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Energy Market-Based Control of Linear Civil Structures

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ABSTRACT

Structural control systems are used to limit structural responses during large external loads such as winds and earthquakes. The semi-active control approach has grown in popularity due to inexpensive control devices that consume little power. Structural control systems of the future will likely be large-scale systems defined by high actuation densities. Decentralized control approaches can be used to control large-scale systems that are too complex for a traditional centralized approach. This paper describes the derivation of a decentralized energy market-based control (EMBC) approach that models the structural control system as a marketplace. The interaction of free-market buyers and sellers results in an optimal allocation of limited control system resources. A 20-story structure is selected as an illustrative example to compare the performance of the EMBC and the centralized linear quadratic regulation (LQR) approaches.

INTRODUCTION

Structural control has rapidly matured over the past decade into a viable means of limiting structural responses to strong winds and earthquakes. To date, well over thirty structures, primarily in Asia, have been constructed with active and semi-active structural control systems installed (Nishitani and Inoue 2001). A structural control system is a complex mechanical system that entails installation of sensors to measure structural responses, actuators to apply forces and a computer (controller) to coordinate the activities of the system including the calculation of control forces based on sensor measurements.

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Three types of structural control systems can be defined: passive, active and semi-active. Passive control is defined by a system entailing the use of passive energy dissipation devices to control the response of a structure without the use of sensors and controllers. Base isolation systems and dampers represent the most popular passive control technologies. Active control is defined by a control system that uses a small number of large actuators for the direct application of control forces. In semi-active control systems, semi-active control devices are used for indirect application of control forces. By changing their energy dissipation properties in real-time, semi-active devices utilize the motion of the system for the generation of control forces.

While success has been attained in applying active control solutions to civil structures, they suffer from some technological limitations. Active control systems are expensive to operate due to their high power demands, often on the order of tens of kilowatts (Symans and Constantinou 1999). Furthermore, active control devices are ill suited for large seismic disturbances because their maximum attainable control forces are not sufficient.

In response to these limitations, semi-active control devices were developed. Small in size and highly reliable, they represent the future of structural control. Since semi-active device do not apply forces directly to a structure, only small amounts of power are required for operation, often on the order of tens of watts (Symans and Constantinou 1999). An additional benefit of semi-active control is the ability to limit structural responses resulting from large seismic disturbances. In 2001, the Kajima-Shizuoka Building, Shizuoka, Japan was constructed with eight semi-active variable dampers, making it the first building to implement semi-active structural control (Kurata et al. 1999).

In observing the trend established by the development of semi-active control systems, innovation will continue to improve upon the design of control devices. In time, semi-active control devices will have smaller form factors, consume less power and become inexpensive. As a result of these developments, adoption of semi-active control in structural engineering will continue to grow. Control systems of the future could potentially depend upon a large number of semi-active devices for control, resulting in a large-scale control problem.

Large-scale control problems require significant computational power for the rapid determination of control forces. Current structural control systems are highly centralized with a single controller employed for the determination of control forces. A centralized controller is generally not suitable for large-scale control problems because control force computations increase at faster than a linear rate with increases in system dimensionality (Lunze 1992). As an alternative to the centralized controller, decentralized control techniques can be considered for adoption. In designing a centralized controller, it is assumed that complete knowledge of the global system (*a priori* information) and complete knowledge of the system's response (*a posteriori* information) are known. For decentralized control, only partial knowledge of the system is known for the appropriate calculation of control forces. The limited information provided to a decentralized controller represents a reduction of computational burden placed on the controllers.

A variety of decentralized control approaches, both *a priori* and *a posteriori* types, can be considered for adoption in a structural control system. *A posteriori* decentralized control reduces the global system to a collection of interrelated subsystems, with each subsystem controlled by a local controller. Approaches similar to the widely used linear quadratic regulator (LQR) have been proposed for the determination of optimal decentralized controllers in an *a posteriori* type decentralized structural control system (Lynch and Law 2002a).

Most recently, the explosion in development of MEMS sensing and actuation systems has resulted in many large-scale control problems. With the reliability of MEMS sensors and actuators lower than conventional counterparts, adaptive and flexible control methods are required with decentralized control solutions most popular. Researchers have explored using free-market concepts as one approach for controlling large-scale MEMS systems (Guenther et al. 1997). By modeling the control system as a free-market economy, where actuators are market buyers and power source are market sellers, an *a priori* decentralized control solution can result. Market-based control (MBC) methods have also been applied for controlling the computational load of microprocessors and for load-balancing in data networks (Clearwater 1996).

Lynch and Law (2002b) have proposed employing market-based control (MBC) for structural systems. To evaluate the feasibility of employing market-based control, a flexible framework was employed, comprised of linear market functions that permit control solutions to be solved algebraically. The optimal MBC solution obtained was effective in reducing responses in structures controlled by semi-active control devices. The scope of this paper is to revisit the MBC derivation in order to develop a rational framework for the approach. Towards that end, naturally occurring measures of energy in the dynamic structural system are used to define the market demand and supply functions. Termed energy market-based control (EMBC), the approach is tested and its control performance is compared to that of the centralized LQR controller. A 20-story benchmark structure with a large-scale control system is used as an illustrative example.

DECENTRALIZED MARKET-BASED CONTROL

The free-market economies are efficient in allocating scarce resources, such as labor and goods, amongst market participants. Free-markets are decentralized in the *a priori* sense because the market mechanisms operate without knowledge of the global system. The historically poor performance of centralized economies underscores the efficiencies of the decentralized marketplaces. The complex workings of a free-market economy can be idealized by the behavior of market institutions such as consumers (buyers), firms (sellers), trade unions and governments. Economists idealize the behavior of market institutions by mathematical functions that encapsulate the behavior and decision process of each institution (Intriligator 1971).

The competitive mechanisms of a free-market can be extended for application to the control paradigm. First, the marketplace is defined by a scarce commodity such as control power, control forces, or control energy, just to name a few potential quantities. In the control marketplace, the role of market buyers and sellers are assumed by system actuators and power sources, respectively. The behavior of buyers is defined by individual utility functions, U_B, that measure the amount of utility derived by the buyer from purchasing the market commodity. Utility is a function of the price per unit commodity, p, the amount of commodity purchased, C_B, and response measures of the dynamic system, y. Similarly, the sellers are governed by individual profit functions, Π_S, that measure the amount of profit derived by the seller from selling the commodity. Profit is modeled as a function of the price per unit commodity, p, and commodity sold, C_S.

The goal of market buyers is to maximize their utility. In doing so, maximization of their utility functions is constrained by limiting the total purchase cost, pC_B, to be less than their instantaneous wealth, W. Maximization of the market sellers' profit functions is constrained by the maximum amount of commodity they possess, C_{MAX}.

$$\begin{aligned}
&\max \Pi_{S1}(C_{S1}, p) \ \text{ subject to } \ C_{S1} \le C_{MAX1} \\
&\max \Pi_{S2}(C_{S2}, p) \ \text{ subject to } \ C_{S2} \le C_{MAX2} \\
&\qquad\qquad \vdots \\
&\max U_{B1}(C_{B1}, p, y_{B1}(t)) \ \text{ subject to } \ pC_{B1} \le W_1 \\
&\max U_{B2}(C_{B2}, p, y_{B2}(t)) \ \text{ subject to } \ pC_{B2} \le W_2
\end{aligned} \tag{1}$$

The simultaneous optimization of the utility and profit functions of buyers and sellers is viewed as a static optimization problem of the decentralized marketplace since time is not explicitly modeled and the market is assumed frozen in time. This is in contrast to dynamic optimization where time is explicitly included in the market models (Chiang 1992). For example, the linear quadratic regulator (LQR) is a solution to a dynamic optimization problem.

Directly resulting from the static optimization of market utility and profit functions are the demand functions of market buyers and the supply functions of market sellers. The marketplace aggregates the demand functions of the individual buyers to obtain the global demand function of the market. In a likewise manner, the market aggregates the supply functions of all sellers to determine the market's global supply function. At each point in time, the demand function and supply function of the market share a point where they intercept. This equilibrium point represents a state of competitive equilibrium that sets the equilibrium price of the commodity. With the equilibrium price found, the static optimization of the marketplace is complete and a transfer of commodity can exist between the market sellers and market buyers. This solution is termed Pareto optimal in the multi-objective optimization sense. Pareto optimal is defined by a market in competitive equilibrium where no market participant can reap the benefits of higher utility or profits without causing harm to other participants when a resource allocation change is made (Mas-Colell, Whinston, and Green 1995).

During an external excitation to the system, the marketplace goes into action with the market re-optimized at each time step for the determination of an efficient control solution. The demand and supply functions change in time, necessitating a re-evaluation of the marketplace at each time step with individual demand and supply functions aggregated and equated. When an equilibrium price is determined, all market buyers purchase their desired commodities and transfer wealth to the market sellers. The amount of commodity purchased by the market buyers (actuators) is converted into control forces to be applied to the structure. After money has been exchanged, the money obtained by the market sellers is evenly distributed back to the market buyers to represent income for their future purchases.

In solving a static optimization problem at each point in time, the resulting control solution of the MBC approach indirectly accounts for changes that occur in the system over time. In some sense, the approach can be viewed as a piece-wise static optimization solution to the dynamic optimization problem. A clear advantage of the MBC solution is its ability to account for changes in system properties as they occur in real-time, resulting in robust control solutions with respect to actuation failures (Lynch 2002).

The earlier derivation, proposed by Lynch and Law (2002b), began with linear demand and supply functions to describe the behavior of market buyers and sellers bidding for the limited commodity of control power, P. While the supply function is held fixed, the demand function is

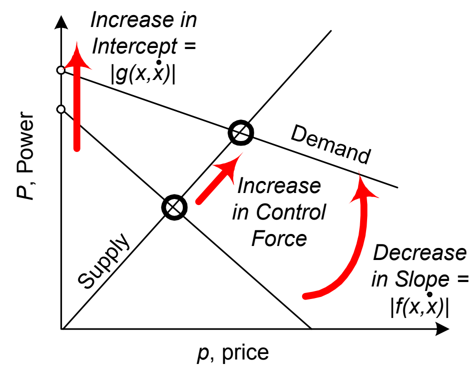


Figure 1 – Behavior of the linear market functions of the MBC control approach

modeled to be responsive to changes in the structural response. With greater responses, the demand function intercept increases while the slope decreases, as illustrated in Fig. 1.

After the equilibrium price of power, p, was determined at each point in time, market buyers use their wealth, W_i, to purchase power. To ensure a means of adjusting the sensitivities of the linear demand and supply functions to changes in the structural response, various constants were included. Specific to the market demand functions, four constants (Q, R, S, and T) were used:

$$P_{D_i} = \left(- \left| \frac{1}{Tx_i + Q\dot{x}_i} \right| p + \left| Rx_i + S\dot{x}_i \right| \right) W_i \tag{2}$$

The variable, x_i, represents the displacement response of the structural degree-of-freedom corresponding to the i^{th} market buyer.

Only one constant (β) is included in the market seller's supply function. The inverse of β represents the constant slope of the market supply function:

$$P_{SUPPLY} = \frac{1}{\beta} p \tag{3}$$

For the marketplace comprised of m market buyers and n market sellers, the equilibrium price of power at each time step can be algebraically determined:

$$p_{eq} = \frac{\sum_{i=1}^{m} W_i \left| Rx_i + S\dot{x}_i \right|}{n/\beta + \sum_{i=1}^{m} W_i / \left| Tx_i + Q\dot{x}_i \right|} \tag{4}$$

This derivation of MBC represents just one of many permissible derivations. The linear marketplace functions were selected to only encapsulate increased demand with increased system response and constant market supply. While easy to implement, the MBC derivation lacked a rational framework and suffers from having to adjust too many tuning constants in order to obtain superior control results. In response to these limitations, the MBC derivation is revisited to provide an energy framework for the derivation of market functions.

ENERGY MARKET-BASED CONTROL

The energy balance of a structural system during a seismic disturbance can easily be derived. First consider the equation of motion of an n degrees-of-freedom structural system subjected to a seismic disturbance and using controls to limit responses that would result:

$$\boldsymbol{M}\ddot{\boldsymbol{y}}(t) + \boldsymbol{C}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) = \boldsymbol{D}\boldsymbol{u}(t) \tag{5}$$

The displacement response vector of the system is $\boldsymbol{x}(t)$, the control forces applied to the system by m actuators are represented by $\boldsymbol{u}(t)$, and the absolute displacement is $\boldsymbol{y}(t)$. The absolute displacement of the system, $\boldsymbol{y}(t)$, is simply the input ground displacement, $x_g(t)$, added to each term of the relative displacement vector, $\boldsymbol{x}(t)$. The mass, damping, and stiffness matrices are n x n in dimension and are denoted by $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$, respectively. It is assumed that the mass, damping and stiffness matrices are symmetric. $\boldsymbol{D}$ is the n x m location matrix for the application of control forces.

Eq. (5) represents the equilibrium balance of forces in the structural system at any point in time. Integrating the forces over the response path from the initial position, $\boldsymbol{x}_o$, to the final position, $\boldsymbol{x}_f$, yields the energy of the balanced system (Wong and Yang 2001).

$$\int_{x_o}^{x_f} \ddot{\boldsymbol{y}}^T \boldsymbol{M} d\boldsymbol{x} + \int_{x_o}^{x_f} \dot{\boldsymbol{x}}^T \boldsymbol{C} d\boldsymbol{x} + \int_{x_o}^{x_f} \boldsymbol{x}^T \boldsymbol{K} d\boldsymbol{x} = \int_{x_o}^{x_f} \boldsymbol{u}^T \boldsymbol{D}^T d\boldsymbol{x} \tag{6}$$

The first term on the left-hand side of Eq. (6) reflects the kinetic energy of the system while the third term represents the strain energy of the system. Both measures of energy are based upon conservative forces and are only dependent upon the current and initial positions of the system. Assuming the system is initially at rest, the kinetic and strain energy of the system can be rewritten and Eq. (6) updated.

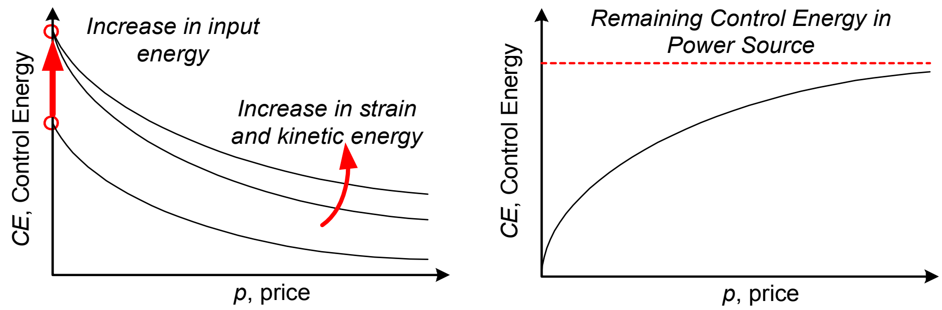
$$\frac{1}{2}\dot{\boldsymbol{y}}^T \boldsymbol{M}\dot{\boldsymbol{y}} + \int_{x_o}^{x_f} \dot{\boldsymbol{x}}^T \boldsymbol{C} d\boldsymbol{x} + \frac{1}{2}\boldsymbol{x}^T \boldsymbol{K}\boldsymbol{x} - \int_{x_o}^{x_f} \boldsymbol{u}^T \boldsymbol{D}^T d\boldsymbol{x} = \int_{x_o}^{x_f} \ddot{\boldsymbol{y}}^T \boldsymbol{M} d\boldsymbol{x}_g \tag{7}$$

The four terms of the left-hand side of Eq. (7) represent respectively, kinetic energy (KE), damping energy (DE), strain energy (SE) and control energy (CE). These energies balance the input energy (IE) resulting from the ground motion as shown on the right-hand side of Eq. (7).

The derivation of energy market-based control (EMBC) is centered upon a marketplace allocating the scarce commodity of control energy. The method begins with the selection of demand and supply functions that reflect measures of energy in the system. The demand and supply functions will each contain a "tuning" constant that can be used to vary their sensitivities.

In the energy marketplace, the scarce commodity of control energy is used to determine the magnitude of control forces applied to the structural system. The form of the demand function is selected to reflect two intentions of the market buyers. First, when the price of control energy is zero, the demand of the market buyer is equal to the input energy of its degree-of-freedom.

Second, the demands of the market buyers asymptotically converge toward zero at infinite



(a) Market Buyer Demand Function ***(b) Market Seller Supply Function***

Figure 2 - Energy market-based control (EMBC) demand and supply functions

prices. To encapsulate these two characteristics, an exponential demand function for the i^{th} market buyer is proposed:

$$CE_i = W_i \left| \ddot{y}_i(t) m_i dx_g \right| e^{\frac{-2p\alpha}{\dot{y}_i^2 m_i + x_i^2 k_i}} \tag{8}$$

The y-axis intercept is equal to the instantaneous input energy of the ground motion at a particular degree-of-freedom multiplied by the market buyer's wealth, W_i. The exponential decay of the demand function is dependent upon the kinetic and strain energy of the system as depicted in the denominator of the exponential term. As the response of the system increases due to greater kinetic and strain energy, the rate of decay decreases. The tuning constant, α, is provided to control the sensitivity of the demand function. Fig. 2(a) illustrates the behavior of the modeled demand function.

The control system's battery sources represent the market sellers whose actions are described by supply functions. Each market seller has in its possession a certain amount of control energy. Again, two observations of the market seller's behavior are required before specifying a suitable supply function. First, if the price of power is set to zero, no market seller is willing to sell. Second, as the price grows to infinity, each market buyer would be willing to sell all of its remaining control energy denoted by L_i. As a result, the following supply function is proposed:

$$CE_i = L_i \left(1 - e^{-\beta p}\right) \tag{9}$$

Eq. (9) provides an origin intercept in addition to an asymptotic convergence to the remaining battery life at very large market prices. The constant β is used to provide a means of adjusting the supply function. Fig. 2(b) presents a graphical interpretation of the market seller supply function.

With the demand and supply functions for all market participants established, the Pareto optimal price at each time step can be readily determined. The aggregate demand function is set equal to the aggregate supply function to determine the competitive equilibrium price of energy for a given time step. A graphical interpretation, as shown in Fig. 3, is the price of control energy of the point where the global demand and supply functions intersect. It can be shown that

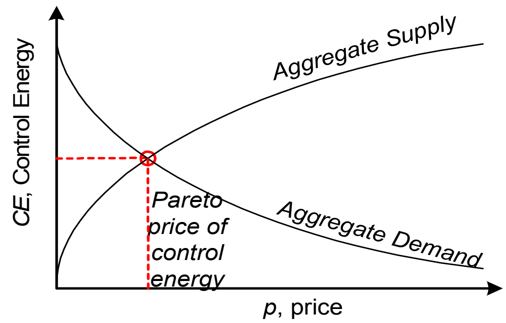


Figure 3 - Determination of the competitive equilibrium price of control energy

this intersection point always exists. The solution represents a Pareto optimal price of control energy for the marketplace.

The amount of control energy that is purchased by each system actuator is used to determine the applied control force. Given the instantaneous control energy purchased by an actuator, the control force u_i can be determined from Eq. (10).

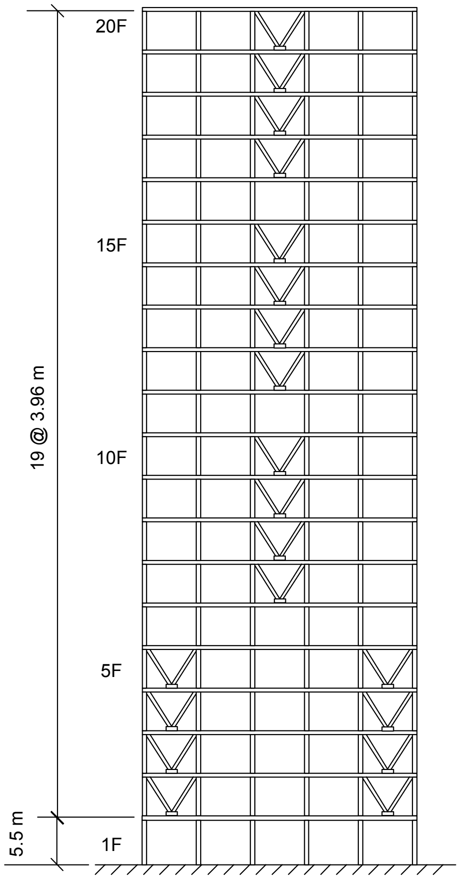
$$CE = u_i \Delta x_i \tag{10}$$

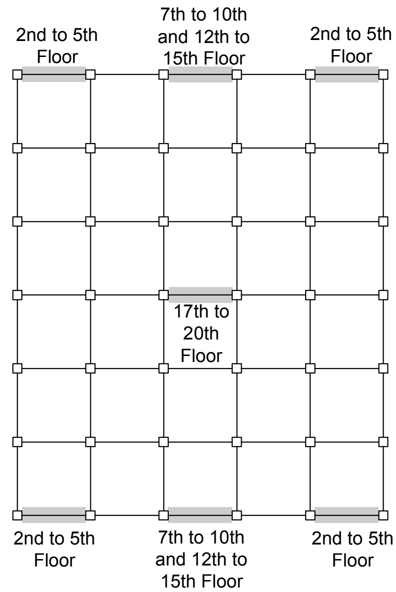
After control energy has been purchased, the energy is subtracted from the system power sources. Similarly, the amount of energy purchased by an actuator times the market price per unit energy determines the amount of wealth removed from each actuator's total wealth. Wealth attained by market sellers is evenly distributed to the market buyers to represent wages for their labor. Therefore, wealth is conservatively maintained by the market buyers.

APPLICATION TO A LARGE-SCALE ANALYTICAL STRUCTURE

A 20-story steel structure, designed for the Southern Los Angeles region as part of the Structural Engineers Association of California's SAC project, is considered (Spencer et al. 1998). The structure represents a realistic large-scale structural system for control. The structural properties of the 20-story benchmark structure are presented in Fig. 4. The structure is modeled as a lumped mass system that experiences elastic responses. Nonlinear (material or geometric) responses are not considered in the analysis. The intention of the analysis is to assess the effectiveness of the EMBC and LQR solutions in a linear structural system.

To control the response of the structure to seismic disturbances, 36 semi-active hydraulic dampers (SHD), designed by Kajima Corporation, Japan, are employed (Kurata et al. 1999). SHD devices are generally attached to the apex of a V- or K-brace and a floor of the structure. The relative motion between stories induces motion in the damper, with a desired control force generated by selecting a damping coefficient of the variable damper. The SHD is capable of a maximum control force of 1,000 kN and a maximum shaft displacement of +/- 6 cm. The coefficient of damping of the device can be adjusted to any value in the range of 1,000 to 200,000 kN-s/m. To power the internal mechanism used to change the coefficient of damping, each SHD consumes 70 Watts of power.





Floor	Seismic Mass (kg)	Story Stiffness (kN/m)
1	1.126×10^6	862,070
2-5	1.100×10^6	862,070
6-11	1.100×10^6	554,170
12-14	1.100×10^6	453,510
15-17	1.100×10^6	291,230
18-19	1.100×10^6	256,460
20	1.170×10^6	171,700

Figure 4 – 20-story SAC benchmark structure

Due to the flexibility of the bracing used to connect the SHD device to the structural system, the bracing and SHD are modeled as a Maxwell damping element (Hatada et al. 2000). A Maxwell element is a spring and dashpot in series whose force, $p(t)$, is determined from the differentiable equation:

$$\dot{p}(t) + \frac{k_{eff}}{c_{SHD}} p(t) = k_{eff}\,\dot{x}(t) \tag{11}$$

The combined stiffness of the bracing element and the inherent stiffness of the damper are combined as k_{eff}.

To excite the structure, three earthquake records are chosen: El Centro (1940 NS) and Taft (1952 NS) represent far field records while Northridge (1994 NS – Sylmar County Hospital) is selected as a near field record. All three earthquakes are scaled so that their absolute peak velocities are 50 cm/sec.

Energy Market-Based Control Solution

An EMBC controller is designed first. The constants for tuning the demand and supply functions of the energy marketplace are set to unity values ($\alpha = 1$ and $\beta = 1$). Each floor of the structure containing actuators represents a market buyer and is provided with an initial wealth of 1,000.

$$\begin{gathered} W_1 = 0; \quad W_2 = W_3 = W_4 = W_5 = 1000; \quad W_6 = 0; \\ W_7 = W_8 = W_9 = W_{10} = 1000; \quad W_{11} = 0; \quad W_{12} = W_{13} = W_{14} = W_{15} = 1000; \\ W_{16} = 0; \quad W_{17} = W_{18} = W_{19} = W_{20} = 1000; \end{gathered} \tag{12}$$

The total control energy provided to the system power source is 8 x 10^{11} J. This amount of reserve control energy should be sufficient for roughly 5 minutes of continuous use by the system's 36 SHD devices.

$$L_T = 8 \;\; x \;\; 10^{11} \;\; \text{J} \tag{13}$$

Linear Quadratic Regulation Solution

A linear quadratic regulation (LQR) controller is designed using the cost function of Eq. (14) where the state space vector $\boldsymbol{X} = \{ \boldsymbol{x}^T \; \dot{\boldsymbol{x}}^T \}^T$.

$$J = \int_0^\infty \left(\boldsymbol{X}^T \boldsymbol{Q} \boldsymbol{X} + \boldsymbol{U}^T \boldsymbol{R} \boldsymbol{U} \right) dt \tag{14}$$

To ensure a stable solution result, the weighting matrices of the LQR cost function, $\boldsymbol{Q}$ and $\boldsymbol{R}$, are selected to be positive definite as presented by Eq. (15):

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{I} & 10\boldsymbol{I} \\ 10\boldsymbol{I} & 100\boldsymbol{I} \end{bmatrix} \text{ and } \boldsymbol{R} = 1x10^{14} [\boldsymbol{I}] \tag{15}$$

Control Results

Fig. 5 plots the maximum absolute interstory drift of the 20-story benchmark structure for the selected seismic disturbances. The EMBC controller exhibits excellent control results with a performance comparable to that of the centralized LQR controller. Therefore, no penalty is incurred by pursuing a decentralized control approach in place of the centralized LQR controller.

CONCLUSION

Decentralized control is proposed for adoption in increasingly complex control systems defined by high actuation densities. In particular, a novel market-based control (MBC) approach has been proposed. MBC showed promising results when applied to large-scale structural

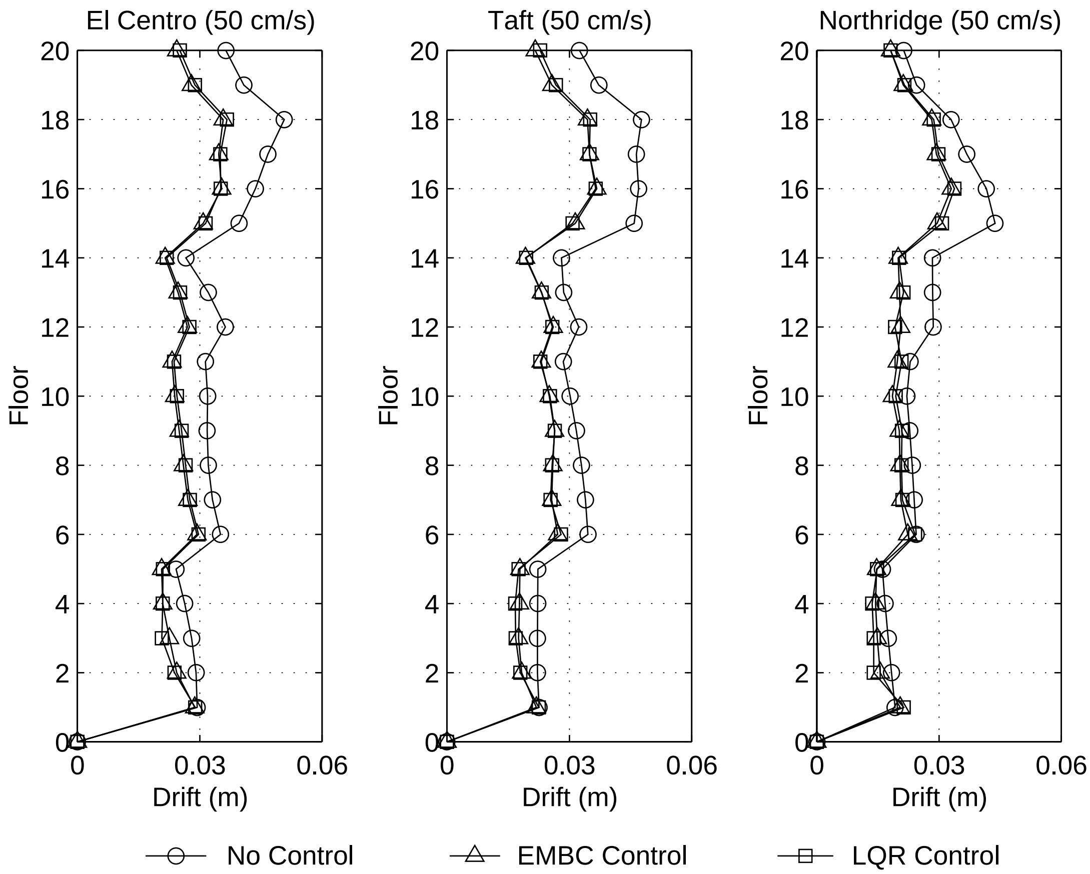


Figure 5 – 20-story benchmark structure maximum interstory drift (LQR, EMBC)

systems (Lynch and Law 200b). In this paper, the MBC derivation has been revisited to provide an energy framework for a new derivation. Termed energy market-based control (EMBC), the energies of a dynamic structural system are used in deriving market demand and supply functions of buyers and sellers, respectively.

The EMBC control solution was implemented in a 20-story large scale structure containing 36 semi-active SHD devices. The structure is excited by two far-field and one near-field seismic records to assist in assessing the performance of the EMBC approach. For comparison purposes, the widely used centralized LQR controller is also implemented. For all seismic records, the EMBC controller yields nearly identical control performances compared to the LQR solution.

Additional work is warranted to explore the robustness qualities of the EMBC approach. As a piecewise static optimization solution, the approach is responsive to changes in the structural system. Preliminary research efforts have shown that the EMBC approach is robust with respect to control device failures (Lynch 2002). The adaptive nature of the EMBC approach can potentially be leveraged to address geometric and material nonlinearities in the structural system.

The stability of EMBC has not been considered in this study. Semi-active control systems are stable in the bounded-input bounded-output (BIBO) sense. However, if EMBC is to be adopted in an active structural control system, or in other control applications, the stability of the approach must be investigated.

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